MODEL OF SUPERCRITICAL TWO-PHASE STEAM-WATER INJECTOR

R. Matysko[1], D. Butrymowicz[*1], W. Angielczyk[2], M. Trela[1], M. Bergander[3]

[1]Institute of Fluid-Flow Machinery of Polish Academy of Sciences, Gdansk, Poland,
[2]Université catholique de Louvain, TERM Division, Louvain-la-Neuve, Belgium
3University of Hartford, 200 Bloomfield Avenue, West Hartford, CT 06117, USA

ABSTRACT. Two-phase steam-water injectors may be applied as a feeding pump device or as a condensing device in many applications. In such injectors steam is a motive fluid while cold water is a secondary fluid. One-dimensional model of two-phase injector has been proposed in the paper. Model is based on two-fluid model of two-phase flow along with a set of closure equations. The results of calculations have been compared with experimental results showing reasonably good agreement.

Keywords: *two-phase flow, injector, two-fluid model, condensation*

INTRODUCTION

Steam –water two-phase injector investigated in the present paper consists of motive steam nozzle, water nozzle, mixing chamber and diffuser. Two different kinds of such injector may be distinguished with respect to the arrangement of the steam nozzle: with a central or outer (annular) nozzle. Figure 1 shows a schematic sketch of the injector with the central arrangement of the steam nozzle which is investigated in the paper. In this case steam is a motive fluid while water is a secondary fluid.

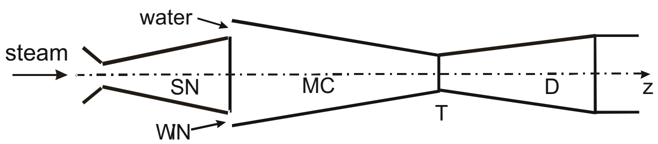

Figure 1. Schematic sketch of steam-water injector:
SN – steam nozzle, MC – mixing chamber, D - diffuser, T- throat, WN – water nozzle.

Having a converging-diverging nozzle shape, the steam is accelerated to the supersonic velocity at the inlet to the mixing chamber. Transfer of heat, momentum (due to the temperature and velocity differences) and mass (due to condensation of steam on water) occur in the mixing chamber. Usually the condensation process is terminated at the throat, followed by a rapid increase of pressure in this region called the shock wave. Then, water is decelerated in the diffuser, which causes a further increase of pressure.

It is seen that the steam injector can be simultaneously used as a water pump and/or a heat exchanger. The exemplary applications of such injector are presented in Fig. 2. The injector can be used as a pumping device in safety passive systems in nuclear power stations (Fig. 2a) in order to

* Corresponding author: D. Butrymowicz
Phone: + (48)-58-6995299, Fax: + (48)-58-3416144
E-mail address: butrym@imp.gda.pl

deliver cooling water to the reactor under breakdown conditions. The other possible application is shown in Fig. 2b where the injectors are used as a feedwater heaters, i.e. as a condensing devices (mixing condensers). Details concerning such applications can be found in papers by Cattadori [5] and Narabayasi [7].

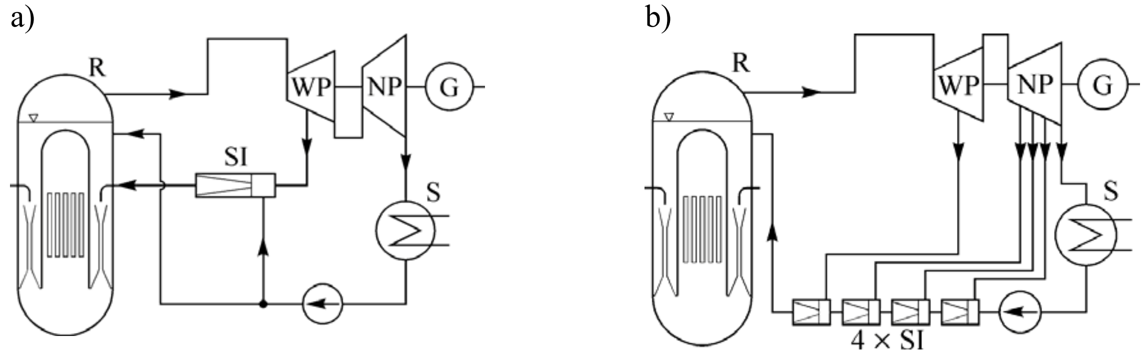

Fig. 2. Exemplary applications of two-phase injectors in nuclear power systems: a) as a cooling water feeding device; b) as a feedwater heating device; R – nuclear reactor; WP – high pressure turbine; NP – low-pressure turbine; S – condenser; G – electric generator.

In many applications of two-phase injector in power generation systems dedicated codes are used to calculate the injector, e.g. RELAP, TRAC, CATHARE – see papers by Pokharna [8]; Carlson [4], Bestion [1]. Various models of two-phase flow are used in order to describe operation of component of power generation systems, mainly: homogenous based on three equations of conservation of mass, energy, and momentum, and two-fluid models based on six equations of conservation for both phases with interphase transport equation, see Ishii [8].

MODEL ASSUMPTIONS

The present paper deals with modelling of two-phase supercritical injector. Two-fluid steady-state model of two-phase flow has been applied for the injector along with closure equations based on the so-called WAHA-3 code equations [11].

The following general assumption has been applied:

- model is steady state;
- all parameters may be averaged through the cross-section area of low, i.e. one dimensional model is applied;
- vapour phase is saturated;
- liquid phase is subcooled.

Moreover, separated flow model have been applied for the closure equations describing the following model quantities: CVM – virtual mass, S – stratification parameter, C_i – interphase shear stress, F_{wall} – flow resistance at the wall. The closure equations of the WAHA-3 code have been used in our calculations [11].

FORMULATION OF THE MODEL

The model of the two-phase stream-water supercritical injector is formulated below. This model consists of the following conservation equations:

a) mass conservation for gas phase:

$$\frac{d}{dz}\left(A\rho_g v_g \alpha\right) = A\Gamma \tag{1}$$

b) mass conservation for liquid phase:

$$\frac{d}{dz}\left(A\rho_l v_l(1-\alpha)\right)=-A\Gamma \tag{2}$$

c) momentum conservation for gas phase:

$$\frac{d\left(A\alpha\rho_g v_g{}^2\right)}{dz}+A\alpha\frac{dp}{dz}+A\cdot C_{VM}\left(v_l\frac{dv_g}{dz}-v_g\frac{dv_l}{dz}\right)+Ap_i\frac{d\alpha}{dz}= \\ =A\alpha\rho_g g\cos\theta-AC_i\left|v_r\right|v_r+A\Gamma v_i-AF_{g,wall} \tag{3}$$

d) momentum conservation for liquid phase:

$$\frac{d\left(A(1-\alpha)\rho_l v_l{}^2\right)}{dz}+A(1-\alpha)\frac{dp}{dz}-A\,C_{VM}\left(v_l\frac{dv_g}{dz}-v_g\frac{dv_l}{dz}\right)-Ap_i\frac{d\alpha}{dz}= \\ =A(1-\alpha)\rho_l g\cos\theta+AC_i\left|v_r\right|v_r-A\Gamma v_i-AF_{l,wall} \tag{4}$$

e) energy conservation for liquid phase:

$$\frac{d\left(A(1-\alpha)\rho_l e_l v_l\right)}{dz}+\frac{d\left(A(1-\alpha)pv_l\right)}{dz}=AQ_{il}-A\Gamma\left(h_l+\frac{v_l^2}{2}\right)+A(1-\alpha)\rho_l g v_l\cos\theta \tag{5}$$

In conservation equation for liquid phase e_l is the sum of specific internal energy and kinetic energy:

$$e_l=h_l-\frac{p}{\rho_l}+\frac{v_l{}^2}{2} \tag{6}$$

It is import ant to note, that vapour is saturated. Because of relation between temperature and pressure under saturated state: $T_s=f(p); T_g=T_s$, there is one independent variable less. Therefore, in this case five conservation equations have been used. The following vector of the state variables is proposed for considered case of the ejector:

$$\varepsilon=\left(\alpha,p,v_g,v_l,h_l\right)^T \tag{7}$$

The further simplifications are possible in the particular case of the ejector. We have assumed that vapour phase is always saturated, therefore:

$$\rho_g=\rho_g(p,x=0)\,. \tag{8}$$

Liquid chase may be treated as incompressible. Taking into consideration above additional assumption the following set of conservations equations have been developed:

a) mass conservation for gas phase:

$$A\rho_g v_g\frac{d\alpha}{dz}+Av_g\alpha\frac{d\rho_g}{dp}\frac{dp}{dz}+A\rho_g\alpha\frac{dv_g}{dz}=A\Gamma-v_g\alpha\rho_g\frac{dA}{dz} \tag{9}$$

b) mass conservation for liquid phase:

$$A\rho_l(1-\alpha)\frac{dv_l}{dz}-Av_l\rho_l\frac{d\alpha}{dz}=-A\Gamma-v_l\rho_l(1-\alpha)\frac{dA}{dz} \tag{10}$$

c) momentum conservation for gas phase:

$$A\alpha\left(v_g^2\frac{d\rho_g}{dp}+1\right)\frac{dp}{dz}+\left(Av_g^2\rho_g+Ap_i\right)\frac{d\alpha}{dz}+\left(2Av_g\alpha\rho_g+AC_{vm}v_l\right)\frac{dv_g}{dz}-AC_{vm}v_g\frac{dv_l}{dz}= \\ Ag\alpha\rho_g\cos\theta-v_g^2\rho_g\alpha\frac{dA}{dz}-AF_{g,wall}-AC_i\left|v_r\right|v_r+Av_i\Gamma \tag{11}$$

d) momentum conservation for liquid phase:

$$A(1-\alpha)\frac{dp}{dz}-(Av_l^2\rho_l+Ap_i)\frac{d\alpha}{dz}-AC_{vm}v_l\frac{dv_g}{dz}+\left(2Av_l\rho_l(1-\alpha)+AC_{vm}v_g\right)\frac{dv_l}{dz}= \\ Ag(1-\alpha)\rho_l\cos\theta-v_l^2\rho_l(1-\alpha)\frac{dA}{dz}-AF_{l,wall}+AC_i\left|v_r\right|v_r-Av_i\Gamma \tag{12}$$

e) energy conservation for liquid phase:

$$A\rho_l(1-\alpha)\left(h_l+\frac{3v_l^2}{2}\right)\frac{dv_l}{dz}-Av_l\rho_l\left(h_l+\frac{v_l^2}{2}\right)\frac{d\alpha}{dz}+Av_l\rho_l(1-\alpha)\frac{dh_l}{dz}=$$
$$Agv_l(1-\alpha)\rho_l\cos\theta-v_l\rho_l(1-\alpha)\left(h_l+\frac{v_l^2}{2}\right)\frac{dA}{dz}+AQ_{il}-A\Gamma\left(h_l+\frac{v_l^2}{2}\right) \tag{13}$$

The above set of equations can be transform into the matrix form:

$$B\frac{d\varepsilon}{dz}=C\,, \tag{14}$$

and it further can be transformer to the following relation:

$$B^{-1}B\frac{\partial\varepsilon}{\partial z}=B^{-1}C\,. \tag{15}$$

The injector may be treated as the momentum exchanger, therefore a key role in the model play the closure equations, especially for variable in momentum conservation equations. The following terms influence on momentum transfer between two phases in the model:

- $\Gamma(v_i-v_k)$ is the momentum transfer between phases at the interface; here Γ is vapour mass condensed. In our case we assume that: $v_i=v_l$ and v_k is the velocity of phase (liquid or vapour).
- $AC_i|v_r|v_r$ describe the frictional stress at the interface. Here: $v_r=v_g-v_l$.
- $A(1-S)C_{VM}\left(v_l\frac{\partial v_g}{\partial z}-v_g\frac{\partial v_l}{\partial z}\right)$ describes momentum transported with virtual mass due to movement of vapour bubbles. Term $(1-S)$ describes influence of flow structure on momentum transfer. The stratification parameter S enables the discussed set of equations is hyperbolic, Drew in [6].
- $Ap_i\frac{\partial\alpha}{\partial z}$ is the term where the so-called interphase pressure is used. In our case the interphase pressure has not a strict physical interpretation and is used in order to ensure the hyperbolic character of the discussed set of equations. For the separated two-phase flows the following relation is proposed by Tiselj in [11]

$$p_i=\alpha(1-\alpha)\Delta\rho g D_h \tag{16}$$

- The stratification parameter S defined by Carlson in [4] and Tiselj in [11] is used in the paper. The parameters C_{VM} and p_i depends on the stratification parameter. For separated flow $S=1$. The consequence of separated flow is diminishing of influence of C_{VM} on momentum transfer as well as increasing of the role of p_i. The following relation is proposed by Tiselj in [11]:

$$S=S_nX_iX_{\rho v}X_\alpha X_{1-\alpha} \tag{17}$$

  where the factors describe influence of various effects on the flow structure:
  S_n - factor of Kelvin-Helmholtz instability;
  X_i - factor of channel inclination;
  $X_{\rho v}$ - factor of very high mass fluxes;
  X_α - factor of very high or very small void fraction;

Condensation heat transfer plays a key role for the operation of the discussed injector. Based on the systematic experimental investigations Trela and Butrymowicz [12] proposed the following correlation describing condensation heat transfer in the injector:

$$Nu = 210.8943 \cdot 10^{-6} \, \mathrm{Re}_v^{0.8-3.09561\Delta-0.134536X} \\ Ja^{-1.86626+8.948750\Delta-0.067246X} \, Oh^{-0.362097-8.86765\Delta-0.211543X} . \quad (18)$$

in the above equation is used dimensionless water feeding gap thickness:

$$\Delta = \frac{\delta}{D_{th}} . \quad (19)$$

The further details of the model are presented in the report of Butrymowicz et al. [3].

RESULTS OF NUMERICAL CALCULATIONS

Two geometry configurations of the injector are presented in Fig. 3. There is a cylindrical throat of the mixing chamber in the case ‘B’ while in the case ‘A’ there is a sharp-edge throat. The case ‘A’ was investigated previously by the authors in paper [13]. The numerical calculations have been carried out for the geometry of the injector investigated in [2]. The calculations of the operation of steam-water injector have been carried out for the operating parameters used in the experiments [2]. The operating parameters for various geometries are given in Table 1.

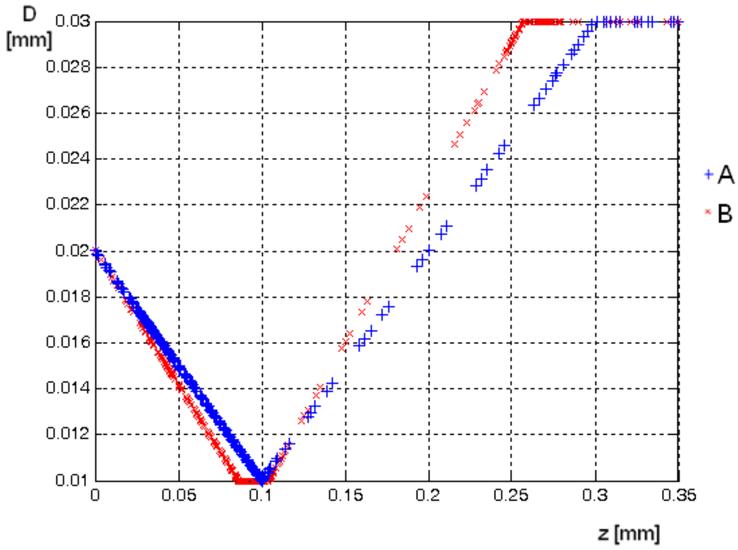

Fig. 3. Geometry of mixing chamber and diffuser with mesh grid of investigated injector. A - injector with sharp-edge throat; B - injector with cylindrical throat

Table 1. Operating parameters for the injector calculations

ejector with sharp-edge throat			ejector with cylindrical throat		
run No.	discharge pressure $[Pa]$	volume rate of liquid $[dm^3/h]$	run No.	discharge pressure $[Pa]$	volume rate of liquid $[dm^3/h]$
A	$p_{out} = 448500$	$\dot{Q}_l = 4895.8$	A	$p_{out} = 492925$	$\dot{Q}_l = 4692$
B	$p_{out} = 349000$	$\dot{Q}_l = 4876.2$	B	$p_{out} = 454552$	$\dot{Q}_l = 4769.6$
C	$p_{out} = 199100$	$\dot{Q}_l = 4000.7$	C	$p_{out} = 388463$	$\dot{Q}_l = 4710.0$
D	$p_{out} = 191700$	$\dot{Q}_l = 3005.8$	D	$p_{out} = 279994$	$\dot{Q}_l = 4777.8$

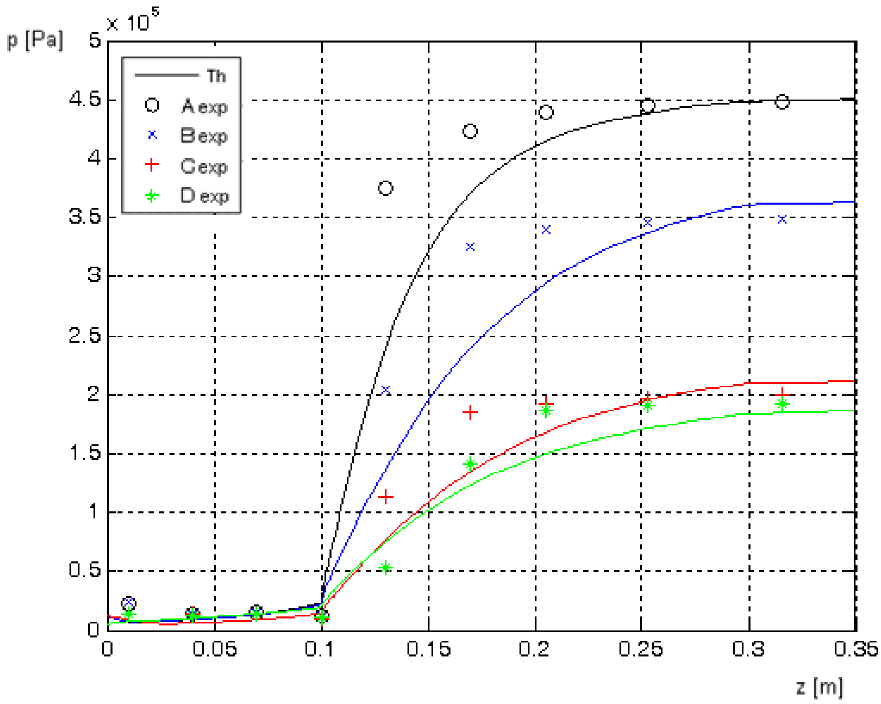

Fig. 4. Comparison of pressure profile calculated (lines) with experimental data for ejector with sharp-edge mixing chamber throat.

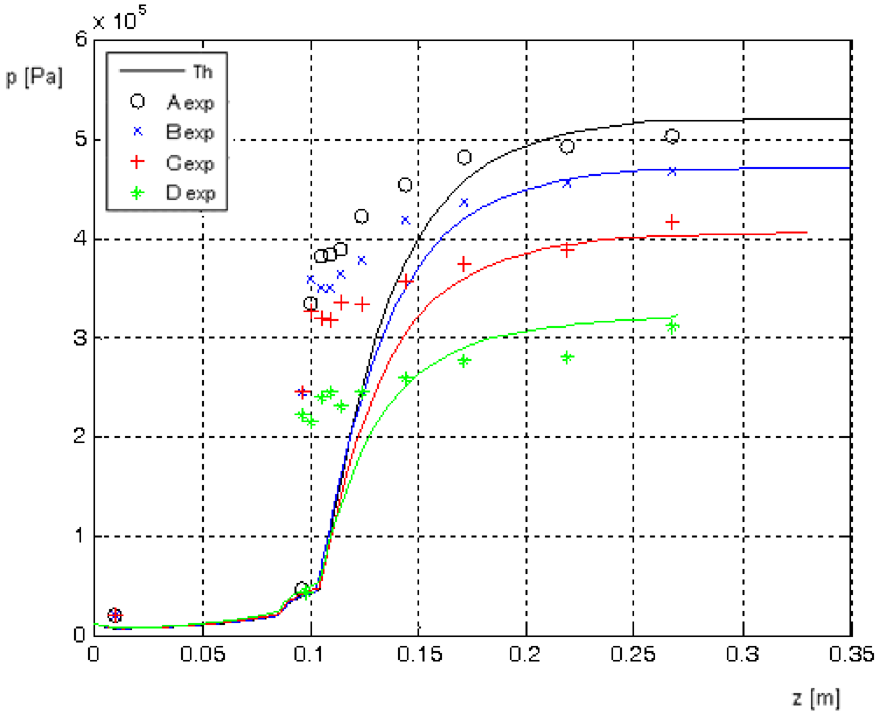

Fig. 5. Comparison of pressure profile calculated (lines) with experimental data for ejector with cylindrical throat of mixing chamber

The calculation results are compared with experimental data in Fig. 4 and 5 for the case of sharp-edge throat and cylindrical throat of mixing chamber, respectively. A reasonably good agreement between calculation results and experiments was shown for the case 'A'. In the case 'B' however, the calculated discharge pressure agrees very well with experiments, there is not very good prediction of the position of condensation shock wave (see Fig. 5). In the case 'B' experiments proved that shock-wave produce relatively higher pressure gradient in comparison with case 'A' which has been also obtained in presented calculations.

CONCLUSIONS

Two-fluid model have been applied to calculate pressure distribution in two-phase injector along with a set of closure equations used in the WAHA-3 code [11]. There is reasonably good agreement between experimental data and calculation results. However, there is a clear need to improve stratification model in order to better describe pressure rise within condensation shock wave, especially in the case of the injector with cylindrical throat of the mixing chamber.

NOMENCLATURE

A	-	surface area cross-section, m^2
B	-	Matrix, see Eq. (14)
c	-	specific heat, J/(kg·K)
C	-	source-terms vector
C_i	-	interfacial friction coefficient
C_{VM}	-	virtual mass coefficient, kg/m^3
D	-	diameter, m
e	-	specific total energy, J/kg
F	-	volume specific wall friction force, N/m^3
g	-	gravity acceleration, kg/s^2
h	-	specific enthalpy, J/kg
h_{fg}	-	specific latent heat of vaporisation, J/kg
Ja	-	Jacob number, $c_{p,l}\left(T_g - T_l\right)/h_{fg}$
Nu	-	Nusselt number $\beta D/\lambda_l$
Oh	-	Ohnesorge number $\mu_l/\sqrt{\sigma D \rho_l}$
p	-	pressure, Pa
Re	-	Reynolds number $\rho v D/\mu$
S	-	stratification factor
v	-	velocity, m/s
Q	-	heat transfer rate per unit of volume [W/m^3]
X	-	Martinelli parameter;
x	-	quality
z	-	spatial coordinate, m

Greek symbols

α	-	void fraction;
β		heat transfer coefficient, W/(m^2·K)
δ	-	liquid gap thickness, m
ε	-	vector of state variables
λ	-	thermal conductivity, W/(m·K)
Γ	-	vapor generation rate, kg/(m^3·s)
θ	-	asimuth angle
μ	-	dynamic viscosity, kg/(m· s)
ρ	-	density, kg/m^3
σ	-	surface tension, N/m

Subsripts

g	-	gas
i	-	interface
l	-	liquid
r	-	relative quantity

s - saturation
th - throat.

REFERENCES

1. Bestion D. (2000) The phase appearance and disappearance in the CATHARE code, *Trends in Numerical and Physical Modeling for Industrial Multiphase Flows*, Cargase, France, pp. 1-10.
2. Buła M., Kwidziński R., Karwacki J., Trela M. (2003) Experimental investigations of distributions of temperature and pressure in stem injector (in Polish), *Technical Report of the Institute of Fluid-Flow Machinery,* No. 3448/2003, Gdansk, Poland.
3. Butrymowicz D., Matysko R., Angielczyk W. (2008) Application of two-fluid model for modelling of supercritital two-phase injector (in Polish), *Technical Report of the Institute of Fluid-Flow Machinery,* Gdansk, Poland.
4. Carlson K. E. et al. (1990) Relap5/Mod3 Code Manual, Vol. 1-7, *Nureg/CR-5535*, EG&G Idaho, Idaho Falls.
5. Cattadori G., Galbiati L., Mazzocchi L., Vanini P. (1995) A single-stage high pressure steam injector for next generation reactors: test results and analysis, *Int. Journal of Multiphase Flow*, Vol. 21, No 4, pp 591-606.
6. Drew D., Cheng L., Lahey R.T. (1979) The analysis of virtual mass effects in two-phase flow *Int J. Multiphase Flows*, Vol. 5, pp. 233-242.
7. Narabayashi T., Ohmori S., Mori M., Asanuma Y., Iwaki C. (2006) Development of multi-stage steam injector for feedwater heaters in simplified nuclear power plant, *JSME Journal*, pp. 368-376.
8. Ishii M., Hibiki T.: Thermo-Fluid dynamics of two-phase flow. *Springer* 2006
9. Pokharna H., Mori M., Ransom V.H. (1997) Regularization of Two-Phase Models: A comparison of Numerical and Differential Approaches. *Journal of Computational Physics* 134, pp. 282-295.
10. Prosperetti A., Satrape J.V.(1990) Stability of two-phase flow models. The IMA volumes in mathematics and its applications, Vol. 26, *Springer-Verlag*, New York.
11. Tiselj I. et al. (2004) WAHA3 Code manual. JSI Report, Ljubljana.
12. Trela M., Butrymowicz D. (2004), Investigations of heat transfer in supersonic steam-water injector, *Proceeding of the Int. Conference on Heat Transfer and Renewable Sources of Energy*, Międzyzdroje, Poland.
13. Butrymowicz D., Matysko R., Angielczyk W., Trela M. (2008) Model of Supercritical Two-phase Steam-water Injector, *5th International Conference on Transport Phenomena in Multiphase Systems*, Białystok, Poland, Vol. 2, pp. 411-415.